PROPERTIES OF LOGARITHMS

$$\log_a 1 = 0 \qquad \log_a a = 1$$

$a^0 = 1$ $\qquad$ $a^1 = a$

Evaluate using the properties of logarithms: ⓐ $\log_8 1$ and ⓑ $\log_6 6$.

$\log_8 1 = 0$

$8^x = 1$

$\log_6 6 = 1$

$6^x = 6$

INVERSE PROPERTIES OF LOGARITHMS

For $a > 0, x > 0$ and $a \neq 1$,

$$a^{\log_a x} = x \qquad \log_a a^x = x$$

$\log_a x = \log_a x$ $\qquad a^x = a^x$

Evaluate using the properties of logarithms: ⓐ $4^{\log_4 9}$ and ⓑ $\log_3 3^5$.

$4^{\log_4 \boxed{9}} = x$

$\log_4 x = \log_4 9$

$x = 9$

$\log_3 3^{\boxed{5}} = ?$

$3^? = 3^5$

$? = 5$

Evaluate using the properties of logarithms: ⓐ $5^{\log_5 15}$ ⓑ $\log_7 7^4$.

If $M > 0, N > 0$, a > 0 and a $\neq 1$, then,

$$\log_a (M \cdot N) = \log_a M + \log_a N$$

The logarithm of a product is the sum of the logarithms.

$\log ab = \log a + \log b$

Use the Product Property of Logarithms to write each logarithm as a sum of logarithms. Simplify, if possible.

ⓐ $\log_3 3x$ ⓑ $\log_2 8xy$

a) $\log_3 3x$

$\log_3 3 + \log_3 x$

$1 + \log_3 x$

b) $\log_2 8xy$

$\log_2 8 + \log_2 x + \log_2 y$

$3 + \log_2 x + \log_2 y$

Use the Product Property of Logarithms to write each logarithm as a sum of logarithms. Simplify, if possible.

ⓐ $\log_9 9x$ ⓑ $\log_3 27xy$

$\log_3 27xy$

$\log_9 9 + \log_9 x$

$\log_3 27 + \log_3 x + \log_3 y$

$1 + \log_9 x$

$3 + \log_3 x + \log_3 y$

If $M > 0, N > 0$,a > 0 and a $\neq 1$, then,

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

The logarithm of a quotient is the difference of the logarithms.

Use the Quotient Property of Logarithms to write each logarithm as a difference of logarithms. Simplify, if possible.

ⓐ $\log_5 \frac{5}{7}$ and ⓑ $\log \frac{x}{100}$

$\log_5 \frac{5}{7}$

$\log_5 5 - \log_5 7$

$1 - \log_5 7$

b) $\log \frac{x}{100}$

$\log x - \log 100$

$\log x - 2$

Use the Quotient Property of Logarithms to write each logarithm as a difference of logarithms. Simplify, if possible.

ⓐ $\log_2 \frac{5}{4}$ ⓑ $\log \frac{10}{y}$

$\log_2 5 - \log_2 4$

$\log_2 5 - 2$

$\log 10 - \log y$

$1 - \log y$

If $M > 0,\ \mathrm{a} > 0,\ \mathrm{a} \neq 1$ and p is any real number then,

$$\log_a M^{p} = p \log_a M$$

The log of a number raised to a power as the product product of the power times the log of the number.

Use the Power Property of Logarithms to write each logarithm as a product of logarithms. Simplify, if possible.

ⓐ $\log_5 4^3$ and ⓑ $\log x^{10}$

$\log_5 4^{3}$

$3\log_5 4$

$\log x^{10}$

$10 \log x$

Use the Power Property of Logarithms to write each logarithm as a product of logarithms. Simplify, if possible.

ⓐ $\log_7 5^4$ ⓑ $\log x^{100}$

PROPERTIES OF LOGARITHMS

If $M > 0,\ N > 0, \mathrm{a} > 0,\ \mathrm{a} \neq 1$ and p is any real number then,

Property	Base a	Base e
	$\log_a 1 = 0$	$\ln 1 = 0$
	$\log_a a = 1$	$\ln e = 1$
Inverse Properties	$a^{\log_a x} = x$ $\log_a a^x = x$	$e^{\ln x} = x$ $\ln e^x = x$
Product Property of Logarithms	$\log_a (M \cdot N) = \log_a M + \log_a N$	$\ln (M \cdot N) = \ln M + \ln N$
Quotient Property of Logarithms	$\log_a \frac{M}{N} = \log_a M - \log_a N$	$\ln \frac{M}{N} = \ln M - \ln N$
Power Property of Logarithms	$\log_a M^p = p \log_a M$	$\ln M^p = p \ln M$

Use the Properties of Logarithms to expand the logarithm $\log_4\left(2x^3y^2\right)$. Simplify, if possible.

$\log_4 2 \cdot x^3 \cdot y^2$

$\log_4 2 + \log_4 x^{3} + \log_4 y^{2}$

$\log_4 2 + 3\log_4 x + 2\log_4 y$

$\frac{1}{2} + 3\log_4 x + 2\log_4 y$

Use the Properties of Logarithms to expand the logarithm $\log_3 \left(7x^5y^3\right)$. Simplify, if possible.

$\log_3 7 + 5\log_3 x + 3\log_3 y$

Use the Properties of Logarithms to expand the logarithm $\log_2 \sqrt[4]{\frac{x^3}{3y^2 z}}$. Simplify, if possible.

$$\log_2 \sqrt[4]{\frac{x^3}{3y^2z}}$$

$$\frac{1}{4}$$

$$\log_2 \left(\frac{x^3}{3y^2z}\right)$$

$$\frac{1}{4}\log_2 \left(\frac{x^3}{3y^2z}\right)$$

$$\frac{1}{4}\left[\log_2 x^3 - \log_2 3y^2z\right]$$

$$\frac{1}{4}\left[3\log_2 x - (\log_2 3 + \log_2 y^2 + \log_2 z)\right]$$

$$\frac{1}{4}\left[3\log_2 x - (\log_2 3 + 2\log_2 y + \log_2 z\right]$$

Use the Properties of Logarithms to expand the logarithm $\log_3 \sqrt[3]{\frac{x^2}{5yz}}$. Simplify, if possible.

$$\log_3\left(\frac{x^2}{5yz}\right)^{1/3}$$

$$\frac{1}{3}\log_3\left(\frac{x^2}{5yz}\right)$$

$$\frac{1}{3}\left[\log_3 x^2 - \log_3 5yz\right]$$

$$\frac{1}{3}\left[2\log_3 x - (\log_3 5 + \log_3 y + \log_3 z)\right]$$

$$\frac{1}{3}\left[2\log_3 x - \log_3 5 - \log_3 y - \log_3 z\right]$$

Use the Properties of Logarithms to condense the logarithm $\log_4 3 \underset{\text{mult}}{+} \log_4 x \underset{\text{Div}}{-} \log_4 y$. Simplify, if possible.

$$\log_4 3x - \log_4 y$$

$$\log_4 \frac{3x}{y}$$

Use the Properties of Logarithms to condense the logarithm $\log_2 5 + \log_2 x - \log_2 y$. Simplify, if possible.